

# Vibrational Analysis Of Cracked Rod Having Circumferential Crack

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**Abstract** -The frequency ratio of torsional vibration of a rod without crack and of rod with crack subjected to torque at the free end for various crack depth and varying crack location is investigated. It is found that even a cracked of small depth is dangerous at the fixed end, also as the crack depth is increases more than 50% of diameter of rod there is a considerable drop in natural frequency of the rod .

A FE model is developed for analysis using CAD software PRO E and analysis is done in ANSYS [12]

## I. INTRODUCTION

One of the most common losses in the structure integrity in mechanical structure is the development and propagation of crack. A crack may propagate from a small imperfection from inside of the material or on the surface it is most likely to appear where stress concentration is high. Some of the areas where crack may generate are fretting corrosion, in case of shrink fitted connection the development of which is also favored by wet and corrosive environment. Thermal stress and thermal shocks are also responsible for crack generation. Behavior of a rotating shaft had been a major concern in the field of fracture mechanics especially for detection of a crack , lots of studies have been done by the researchers.

The dynamic analysis of a rotor system considering a slant crack in shaft was investigated by A. S. Shekhar and P . Balaji Prashad [2], to asses the flexural vibration of a shaft having a slant crack that has resulted from fatigue of the shaft due to the torsinal momentum . A flexible matrix for the slant crack and later a stiffness matrix of a slant crack has been developed to be used subsequently in the FEA analysis of the rotor-bearing system, they had concluded. A general trend of a reduction in the eigenfrequencies of all of the modes with an increase in crack depth has been observed this behavior is similar to the case of the transverse crack.

The dynamic behavior analysis of cracked rotor has been investigated by Oh Sung and Md S Gadala [3], in this study they had presented a additional slop in crack breathing and is expressed explicitly in the equation of motion as one of the input to produce bending moment at the crack position. Verification analysis is carried out for simple rotor model similar to those found in literature. It is shown that the region on crack front line having the dominant stress intensity factor moves from central area to both ends as the crack depth increases. The free vibration analysis of uniform and steeped beam with circular cross section is studied by Murat Kisa and M Arif Gurel [5]. In this approach in which FEM and

component mode synthesis method are used together , the beam is detached in two parts in two section from crack region . These subtract are joined by using the flexible matrix taking in to account the interaction force is derived by virtue of fracture mechanics theory as the inverse of the compliance matrix found with appropriate stress intensity factor and strain energy release rate expression .

Numerical results shows a good agreement with the results of the other available studies , it also shows that the crack location and size can notably influence the modal features i,e natural frequency and mode shapes. D.Y Zheng and N.J Kessissoglou [4] has investigated the free vibration analysis of the cracked beam by finite element method. The natural frequency and mode shapes are obtained using FEM the local flexibility condition at the crack location. They had concluded that by considering additional flexibility matrix instead of the local additional flexibility matrix, more accurate natural frequency of cracked beam are obtained. Myoung-Hwan Choi , Heung-Seok Kang , Kyung-Ho Yoon , Kee-Nam Song , Youn-Ho Jung [6] has investigated the behavior of dumpy fuel rod supported by a spacer grid.

The fuel rod have vibration behavior due to the flow of coolant. In this a model testing and a FE model is generated using ABAQUS on a dumpy fuel rod and natural frequency and mode shape are compared according to the model accuracy criteria value where the New Doublet Spacer Grid considering contact phenomena give better result than the optimized H type. They had concluded the excitation force showing the displacement of about 0.2 mm in the experiment is 0.5 N for the OHT SG and 0.75 N for the ND SGs, respectively.

In this case, the experimental natural frequencies of the fuel rod in air is 32.44 Hz for the OHT SGs and 47.04 Hz for the ND ones, and the frequency of the fuel rod with ND SG is higher by 15 Hz than that of the case of the rod with OHT SGs. For both SGs, natural frequencies under water decrease by about 5 % due to the added mass effects of the fluid. The experimental mode shapes have relatively large differences with those of the FE analysis at the 2nd and 4<sup>th</sup> frequencies showing the 2nd bending mode. Especially, for the fuel rod with ND SGs the discrepancy of the mode for the 2nd span is large, and the 4th mode in the experiment can't be obtained. It is believed that the excitation force distorts the experimental mode of the 2nd span showing the 2nd bending mode. A comparison of mode shapes and MACs between experimental and FEA results show that the results are good and the FE model can be reliable for the fuel rod with OHT SG.

Elastic-Plastic Vibration analysis of a rod is done by Tetsuhiko Miyoshi [7], author had developed an unique weak form of a mathematical model which represent the elastic plastic vibration of a straight uniform rod submitted to longitudinal impact. Jorg Wauer [8] had modeled and formulated the equation of cracked rotating shaft for this author had studied the dynamics of a rotating Timoshenko Shaft which is also flexible in extension and torsion. By above proposed method author concluded study can be focused on the crack effect without being disturbed by secondary consideration, also the procedure is also discussed for torsional vibration of a crack shaft with circumferential crack.

Closed form Solution of the Natural Frequencies and Mode Shapes of a Tapered Torsional Shaft Clamped at One End is presented by Soon-Jo Chung , Marthinus C. van Schoor [9]. The analytical exact solutions for the natural frequencies and mode shapes of clamped-free torsional rods with linearly varying circular cross-sections are obtained and proven to agree with the numerical predictions by the finite element method.

The dynamic stiffness matrix method in forced vibration analysis of multiple cracked beam is given by N. T. KHIEM , T. V. LIEN [10], the dynamic behaviour of a beam with numerous transverse cracks is studied. Based on the equivalent rotational spring model of crack and the transfer matrix for beam, the Dynamic stiffness matrix method has been developed for spectral analysis of forced vibration of a multiple cracked beam. As a particular case, when the excitation frequency is close to zero, the solution for static response of beam with an arbitrary number of cracks has been obtained exactly in an analytical form. In general case, the effect of crack number and depth on the dynamic response of beam was analyzed numerically.

As fatigue is the major factor of crack growth in the rotating shaft so the effect of steady torsion in fatigue crack growth is studied by M. Fonte,, L. Reis , F. Romeiro , B. Li , M. Freitas [11]. In this an analysis of the influence of steady torsion loading on fatigue crack growth rates in shafts is presented for short as well as long cracks The short crack growth rates obtained are compared with long crack growth rates. Results have shown a significant reduction of the crack growth rates when a steady torsion Mode III is superimposed to cyclic Mode I. The torsional effect on the shaft had also been studied by T.G Chondros [13] by variational formulation under this study the author has developed a variational formulation for the torsional vibration of a cylindrical shaft with a circumferential crack. The work is compared with existing methods.

Vibration analysis and diagnosis of cracked shaft is done by T. C. TSAI AND Y. Z. WANG [14]. In this a diagnostic method of determining the position and size of a transverse open crack on a stationary shaft without disengaging it from the machine system is investigated. The crack is modeled as a joint of a local spring [To obtain the dynamic characteristics of a stepped shaft and a multidisc shaft the transfer matrix method is employed on the basis of Timoshenko beam theory.

It can be combined with beam segments to derive the frequency equation for the assembly and is then solved for the frequency as well as the corresponding mode shape of the cracked shaft. Verification of this approach by comparison with some already existing published experimental data is presented. The position of the crack can be predicted by comparing the fundamental mode shapes of the shaft with and without a crack .Furthermore the depth of the crack can be obtained by the change of natural frequency of the shaft with and without a crack.

## II. FE ANALYSIS OF THE VIBRATING CRACKED ROD

The FE formulation of an Euler-Bernoulli cracked beam using 3-D solid elements, leads to a system of linear algebraic equations of the form.

$$[M]^S \{ \ddot{q} \}^S + [C]^S \{ \dot{q} \}^S + [K]^S \{ q \}^S = \{ f \}^S$$

where  $[M]^S$ ,  $[C]^S$  and  $[K]^S$  are the mass, damping and stiffness matrices for the vibrating system and  $q(t)$  the response of the vibrating cracked rod of Fig. 1 in a stationary coordinate system. The damping part of equation is neglected, since undammed vibration is considered here. For the solution of Eq. (21), the developed liberalized 3-D FE model of the cracked rod is shown.

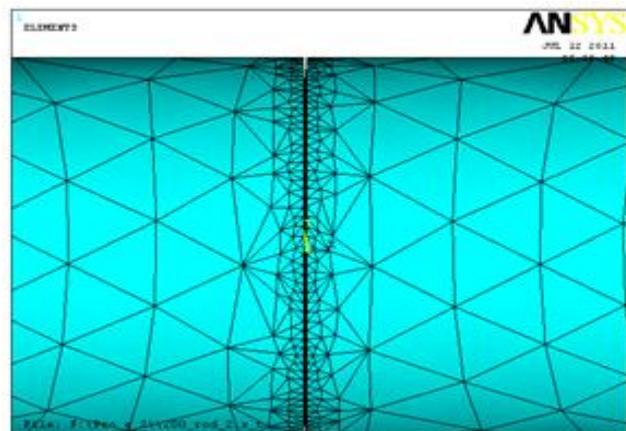


Fig 1. FE model of cracked rod used for analysis

The FE mesh of the considered crack rod is developed using the FE software ANSYS [12] and SOLID92 which has a quadratic displacement behavior. The element is defined by ten nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The cracked surface is modeled as a notch in the cross section. Contact elements are not used in the present model, therefore, contact or friction between the crack faces is not taken into account. The solution of the modal value problem, using the developed FE model, has revealed the extension, bending and twisting modes, as well as their interaction, at present torsional natural frequencies are investigated. The various results of FE mesh are compared with respect to the numerical results for various

crack depth ratio has been studied. Further the position of crack is varied with respect to fixed end of the rod to find out the location of crack where it would be most dangerous.

### III. RESULT

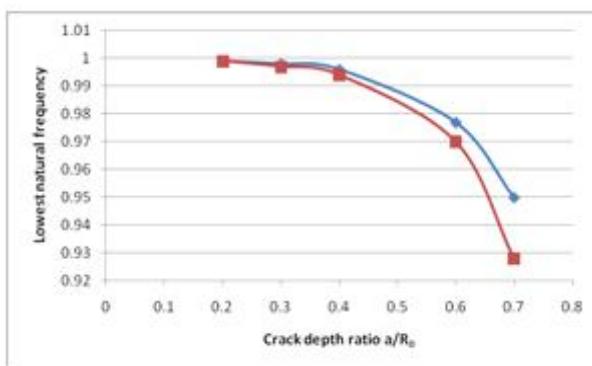


Fig 2. Lowest natural frequency drop for increasing crack depth of the rod in FE analysis. Rod radius to length ratio  $R_0/L_0 = 0.0071$ ;  $R_0/L_0 = 0.0102$

In above figure lowest natural frequencies of cracked rod with different radius to length ratio are compared and it was found as the crack depth increases there is a considerable drop in the natural frequencies of rod

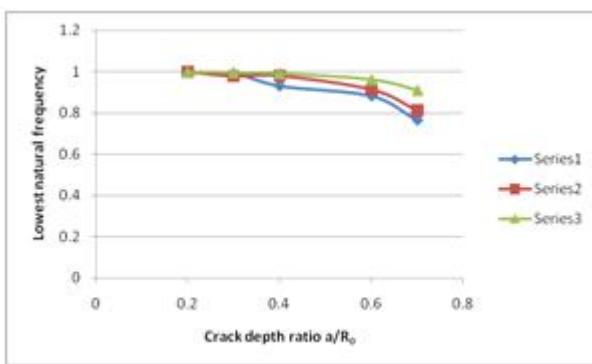


Fig 3. Lowest natural frequency drop for increasing crack depth ratio and at various distance from supported end. Series 1 20mm with  $R_0/L_0 = 0.0071$ ; series 2 40mm; series 3 80mm

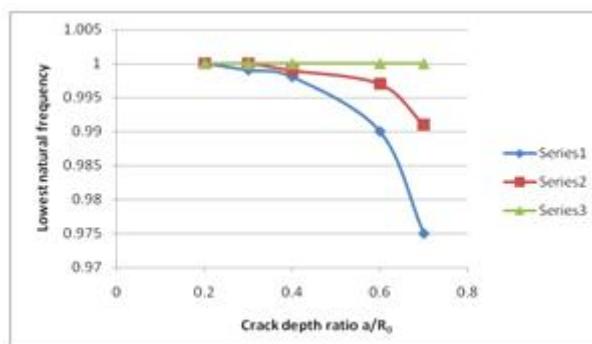


Fig 4. Lowest natural frequency drop for increasing crack depth ratio and at various distance from supported end. Series 1 120mm with  $R_0/L_0 = 0.0071$ ; series 2 140mm; series 3 180mm

In above two fig lowest natural frequencies of cracked rod are compared when length is varied from fixed end to the free end, it is observed that cracked even with small depth are dangerous when it is at the supported end, the radius to length ratio of the rod is 0.0071

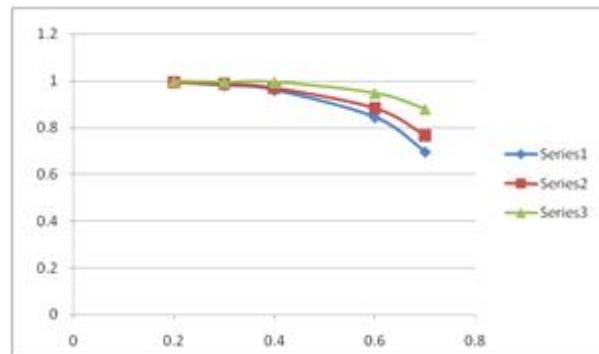


Fig 5. Lowest natural frequency drop for increasing crack depth ratio and at various distance from supported end. Series 1 20mm with  $R_0/L_0 = 0.0102$ ; series 2 40mm; series 3 80mm

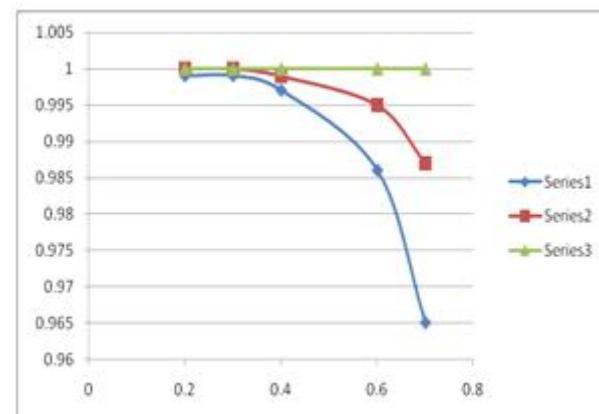


Fig 6. Lowest natural frequency drop for increasing crack depth ratio and at various distance from supported end. Series 1 120mm with  $R_0/L_0 = 0.0102$ ; series 2 140mm; series 3 180mm

In above two fig lowest natural frequencies of cracked rod are compared when length is varied from fixed end to the free end, it is observed that cracked even with small depth are dangerous when it is at the supported end, the radius to length ratio of the rod is 0.0102

### CONCLUSION

In this paper, a parametric 3-D FE model using 3-D solid element was employed for the analysis of the cracked rod behavior. This model is based on the commercial code ANSYS [12] was used for the numerical analysis of dynamic response of the cracked rod. The model is modified accordingly in order to analyses different rod geometrical configuration and varying crack location and depth. Numerical results are compared with the results of literature. The 3-D solid FE model results provide good agreement with the results of the literature. Careful observation of the behavior of these damage models can lead to extension of their utility for detection of practical engineering importance in the area of vibration and fault detection of cylindrical shaft and rotors

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